PROBLEM 5-10

Statement: Design a linkage to carry the body in Figure P5-1 through the three positions P_1, P_2 and P_3 at the angles shown in the figure. Use analytical synthesis without regard for the fixed pivots shown. Use the free choices given below.

Given: Coordinates of the points P_1 and P_2 with respect to P_1:

$P_{1x} := 0.0$ $\quad P_{1y} := 0.0$ $\quad P_{2x} := -1.236$ $\quad P_{2y} := 2.138$

$P_{3x} := -2.500$ $\quad P_{3y} := 2.931$

Angles made by the body in positions 1, 2 and 3:

$\theta_{P1} := 210\cdot deg$ $\quad \theta_{P2} := 147.5\cdot deg$ $\quad \theta_{P3} := 110.2\cdot deg$

Free choices for the **WZ** dyad :

$\beta_2 := 30.0\cdot deg$ $\quad \beta_3 := 60.0\cdot deg$

Free choices for the **US** dyad :

$\gamma_2 := -10.0\cdot deg$ $\quad \gamma_3 := 25.0\cdot deg$

Two argument inverse tangent

$$atan2(x,y) := \begin{cases} \text{return } 0.5\cdot\pi \text{ if } x = 0 \wedge y > 0 \\ \text{return } 1.5\cdot\pi \text{ if } x = 0 \wedge y < 0 \\ \text{return } atan\left(\left(\frac{y}{x}\right)\right) \text{ if } x > 0 \\ atan\left(\left(\frac{y}{x}\right)\right) + \pi \text{ otherwise} \end{cases}$$

Solution: See Figure P5-1 and Mathcad file P0510.

1. Determine the magnitudes and orientation of the position difference vectors.

$p_{21} := \sqrt{P_{2x}^2 + P_{2y}^2}$ $\quad p_{21} = 2.470$ $\quad \delta_2 := atan2(P_{2x}, P_{2y})$ $\quad \delta_2 = 120.033\ deg$

$p_{31} := \sqrt{P_{3x}^2 + P_{3y}^2}$ $\quad p_{31} = 3.852$ $\quad \delta_3 := atan2(P_{3x}, P_{3y})$ $\quad \delta_3 = 130.463\ deg$

2. Determine the angle changes of the coupler between precision points.

$\alpha_2 := \theta_{P2} - \theta_{P1}$ $\quad \alpha_2 = -62.500\ deg$

$\alpha_3 := \theta_{P3} - \theta_{P1}$ $\quad \alpha_3 = -99.800\ deg$

3. Evaluate terms in the **WZ** coefficient matrix and constant vector from equations 5.26 and form the matrix and vector:

$A := \cos(\beta_2) - 1$ $\quad B := \sin(\beta_2)$ $\quad C := \cos(\alpha_2) - 1$

$D := \sin(\alpha_2)$ $\quad E := p_{21}\cdot\cos(\delta_2)$ $\quad F := \cos(\beta_3) - 1$

$G := \sin(\beta_3)$ $\quad H := \cos(\alpha_3) - 1$ $\quad K := \sin(\alpha_3)$

$L := p_{31}\cdot\cos(\delta_3)$ $\quad M := p_{21}\cdot\sin(\delta_2)$ $\quad N := p_{31}\cdot\sin(\delta_3)$

$$AA := \begin{pmatrix} A & -B & C & -D \\ F & -G & H & -K \\ B & A & D & C \\ G & F & K & H \end{pmatrix} \qquad CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix} \qquad \begin{pmatrix} W_{1x} \\ W_{1y} \\ Z_{1x} \\ Z_{1y} \end{pmatrix} := AA^{-1} \cdot CC$$

The components of the **W** and **Z** vectors are:

$W_{1x} = 2.920$ $\quad W_{1y} = 1.720$ $\quad Z_{1x} = -0.756$ $\quad Z_{1y} = -0.442$

$\theta := atan2(W_{1x}, W_{1y})$ $\quad \theta = 30.493\ deg$ $\quad \phi := atan2(Z_{1x}, Z_{1y})$ $\quad \phi = 210.303\ deg$

The length of link 2 is: $w := \sqrt{\left(W_{1x}^2 + W_{1y}^2\right)}$, $\quad w = 3.389$

The length of vector **Z** is: $z := \sqrt{\left(Z_{1x}^2 + Z_{1y}^2\right)}$, $\quad z = 0.876$

4. Evaluate terms in the **US** coefficient matrix and constant vector from equations 5.31 and form the matrix and vector:

$A' := \cos(\gamma_2) - 1$ $\quad B' := \sin(\gamma_2)$ $\quad C := \cos(\alpha_2) - 1$

$D := \sin(\alpha_2)$ $\quad E := p_{21} \cdot \cos(\delta_2)$ $\quad F' := \cos(\gamma_3) - 1$

$G' := \sin(\gamma_3)$ $\quad H := \cos(\alpha_3) - 1$ $\quad K := \sin(\alpha_3)$

$L := p_{31} \cdot \cos(\delta_3)$ $\quad M := p_{21} \cdot \sin(\delta_2)$ $\quad N := p_{31} \cdot \sin(\delta_3)$

$$AA := \begin{pmatrix} A' & -B' & C & -D \\ F' & -G' & H & -K \\ B' & A' & D & C \\ G' & F' & K & H \end{pmatrix} \qquad CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix} \qquad \begin{pmatrix} U_{1x} \\ U_{1y} \\ S_{1x} \\ S_{1y} \end{pmatrix} := AA^{-1} \cdot CC$$

The components of the **U** and **S** vectors are:

$U_{1x} = -1.009$ $\quad U_{1y} = 2.693$ $\quad S_{1x} = -0.792$ $\quad S_{1y} = -2.418$

$\sigma := atan2(U_{1x}, U_{1y})$ $\quad \sigma = 110.545\ deg$ $\quad \psi := atan2(S_{1x}, S_{1y})$ $\quad \psi = 251.875\ deg$

The length of link 4 is: $u := \sqrt{\left(U_{1x}^2 + U_{1y}^2\right)}$, $\quad u = 2.875$

The length of vector **S** is: $s := \sqrt{\left(S_{1x}^2 + S_{1y}^2\right)}$, $\quad s = 2.544$

5. Solve for links 3 and 1 using the vector definitions of **V** and **G**.

Link 3: $V_{1x} := Z_{1x} - S_{1x}$ $\quad V_{1x} = 0.036$

$V_{1y} := Z_{1y} - S_{1y}$ $\quad V_{1y} = 1.976$

$\theta_3 := atan2(V_{1x}, V_{1y})$ $\quad \theta_3 = 88.968\ deg$

$v := \sqrt{V_{1x}^2 + V_{1y}^2}$ $v = 1.977$

Link 1: $G_{1x} := W_{1x} + V_{1x} - U_{1x}$ $G_{1x} = 3.965$

$G_{1y} := W_{1y} + V_{1y} - U_{1y}$ $G_{1y} = 1.003$

$\theta_1 := atan2(G_{1x}, G_{1y})$ $\theta_1 = 14.202\ deg$

$g := \sqrt{G_{1x}^2 + G_{1y}^2}$ $g = 4.090$

6. Determine the initial and final values of the input crank with respect to the vector **G**.

$\theta_{2i} := \theta - \theta_1$ $\theta_{2i} = 16.291\ deg$

$\theta_{2f} := \theta_{2i} + \beta_3$ $\theta_{2f} = 76.291\ deg$

7. Define the coupler point with respect to point A and the vector **V**.

$r_p := z$ $\delta_p := \phi - \theta_3$

$r_p = 0.876$ $\delta_p = 121.335\ deg$

8. Locate the fixed pivots in the global frame using the vector definitions in Figure 5-2.

$O_{2x} := -z \cdot cos(\phi) - w \cdot cos(\theta)$ $O_{2x} = -2.164$

$O_{2y} := -z \cdot sin(\phi) - w \cdot sin(\theta)$ $O_{2y} = -1.278$

$O_{4x} := -s \cdot cos(\psi) - u \cdot cos(\sigma)$ $O_{4x} = 1.801$

$O_{4y} := -s \cdot sin(\psi) - u \cdot sin(\sigma)$ $O_{4y} = -0.274$

9. Determine the rotation angle of the fourbar frame with respect to the global frame (angle from the global X axis to the line O_2O_4.

$\theta_{rot} := atan2\left[\left(O_{4x} - O_{2x}\right), \left(O_{4y} - O_{2y}\right)\right]$ $\theta_{rot} = 14.202\ deg$

10. Determine the Grashof condition.

```
Condition(a, b, c, d) := | S ← min(a, b, c, d)
                         | L ← max(a, b, c, d)
                         | SL ← S + L
                         | PQ ← a + b + c + d − SL
                         | return "Grashof"  if SL < PQ
                         | return "Special Grashof"  if SL = PQ
                         | return "non-Grashof"  otherwise
```

$Condition(g, u, v, w) =$ "Grashof"

11. **DESIGN SUMMARY**

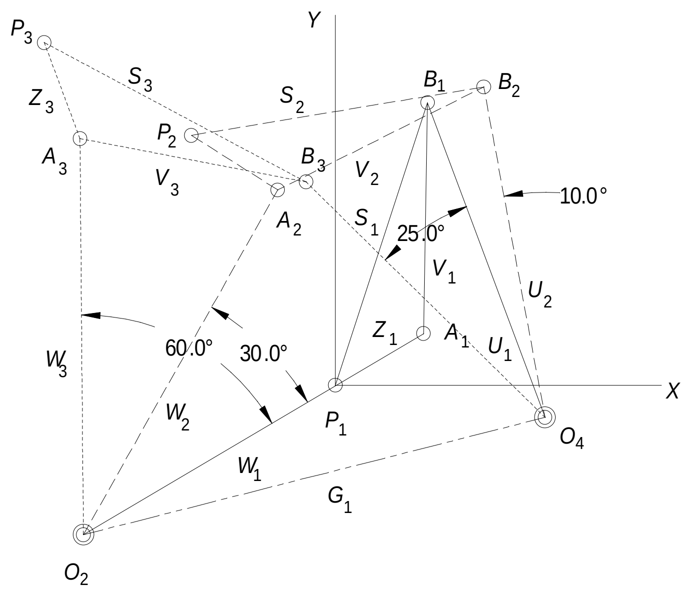
Link 2:	$w = 3.389$	$\theta = 30.493\ deg$
Link 3:	$v = 1.977$	$\theta_3 = 88.968\ deg$
Link 4:	$u = 2.875$	$\sigma = 110.545\ deg$
Link 1:	$g = 4.090$	$\theta_1 = 14.202\ deg$
Coupler:	$r_p = 0.876$	$\delta_p = 121.335\ deg$

Crank angles:

$\theta_{2i} = 16.291\ deg$

$\theta_{2f} = 76.291\ deg$

12. Draw the linkage, using the link lengths, fixed pivot positions, and angles above, to verify the design.



PROBLEM 5-12

Statement: Design a linkage to carry the body in Figure P5-2 through the two positions P_1 and P_2 at the angles shown in the figure. Use analytical synthesis without regard for the fixed pivots shown. Use the free choices given below.

Given: Coordinates of the points P_1 and P_2 with respect to P_1:

$P_{1x} := 0.0$ $\quad P_{1y} := 0.0$ $\quad P_{2x} := 1.903$ $\quad P_{2y} := 1.347$

Angles made by the body in positions 1 and 2:

$\theta_{P1} := 101.0 \cdot deg$ $\quad \theta_{P2} := 62.0 \cdot deg$

Free choices for the **WZ** dyad :

$z := 2.000$ $\quad \beta_2 := 30.0 \cdot deg$ $\quad \phi := 150.0 \cdot deg$

Free choices for the **US** dyad :

$s := 3.000$ $\quad \gamma_2 := 40.0 \cdot deg$ $\quad \psi := -50.0 \cdot deg$

Two argument inverse tangent

$$atan2(x, y) := \begin{cases} return\ 0.5 \cdot \pi & if\ x = 0 \wedge y > 0 \\ return\ 1.5 \cdot \pi & if\ x = 0 \wedge y < 0 \\ return\ atan\left(\left(\frac{y}{x}\right)\right) & if\ x > 0 \\ atan\left(\left(\frac{y}{x}\right)\right) + \pi & otherwise \end{cases}$$

Solution: See Figure P5-2 and Mathcad file P0512.

1. Note that this is a two-position motion generation (MG) problem because the output is specified as a complex motion of the coupler, link 3. Because of the data given in the hint, the second method of Section 5.3 will be used here.

2. Define the position vectors $\mathbf{R}_1$ and $\mathbf{R}_2$ and the vector $\mathbf{P}_{21}$ using Figure 5-1 and equation 5.1.

$$R1 := \begin{pmatrix} P_{1x} \\ P_{1y} \end{pmatrix} \qquad R2 := \begin{pmatrix} P_{2x} \\ P_{2y} \end{pmatrix} \qquad \begin{pmatrix} P_{21x} \\ P_{21y} \end{pmatrix} := R2 - R1 \qquad P_{21x} = 1.903$$

$$P_{21y} = 1.347$$

$$p_{21} := \sqrt{P_{21x}^2 + P_{21y}^2} \qquad p_{21} = 2.331$$

3. From the trigonometric relationships given in Figure 5-1, determine α_2 and δ_2.

$$\alpha_2 := \theta_{P2} - \theta_{P1} \qquad \alpha_2 = -39.000\ deg$$

$$\delta_2 := atan2\left(P_{21x}, P_{21y}\right) \qquad \delta_2 = 35.292\ deg$$

4. Solve for the **WZ** dyad using equations 5.8.

$$Z_{1x} := z \cdot cos(\phi) \qquad Z_{1x} = -1.732 \qquad Z_{1y} := z \cdot sin(\phi) \qquad Z_{1y} = 1.000$$

$$A := cos(\beta_2) - 1 \qquad A = -0.134 \qquad D := sin(\alpha_2) \qquad D = -0.629$$

$B := sin(\beta_2)$ $B = 0.500$ $E := p_{21} \cdot cos(\delta_2)$ $E = 1.903$

$C := cos(\alpha_2) - 1$ $C = -0.223$ $F := p_{21} \cdot sin(\delta_2)$ $F = 1.347$

$$W_{1x} := \frac{A \cdot (-C \cdot Z_{1x} + D \cdot Z_{1y} + E) + B \cdot (-C \cdot Z_{1y} - D \cdot Z_{1x} + F)}{-2 \cdot A} \qquad W_{1x} = 0.452$$

$$W_{1y} := \frac{A \cdot (-C \cdot Z_{1y} - D \cdot Z_{1x} + F) + B \cdot (C \cdot Z_{1x} - D \cdot Z_{1y} - E)}{-2 \cdot A} \qquad W_{1y} = -1.896$$

$$w := \sqrt{W_{1x}^2 + W_{1y}^2} \qquad w = 1.949$$

$$\theta := atan2(W_{1x}, W_{1y}) \qquad \theta = -76.607 \, deg$$

5. Solve for the **US** dyad using equations 5.12.

$S_{1x} := s \cdot cos(\psi)$ $S_{1x} = 1.928$ $S_{1y} := s \cdot sin(\psi)$ $S_{1y} = -2.298$

$A := cos(\gamma_2) - 1$ $A = -0.234$ $D := sin(\alpha_2)$ $D = -0.629$

$B := sin(\gamma_2)$ $B = 0.643$ $E := p_{21} \cdot cos(\delta_2)$ $E = 1.903$

$C := cos(\alpha_2) - 1$ $C = -0.223$ $F := p_{21} \cdot sin(\delta_2)$ $F = 1.347$

$$U_{1x} := \frac{A \cdot (-C \cdot S_{1x} + D \cdot S_{1y} + E) + B \cdot (-C \cdot S_{1y} - D \cdot S_{1x} + F)}{-2 \cdot A} \qquad U_{1x} = 0.924$$

$$U_{1y} := \frac{A \cdot (-C \cdot S_{1y} - D \cdot S_{1x} + F) + B \cdot (C \cdot S_{1x} - D \cdot S_{1y} - E)}{-2 \cdot A} \qquad U_{1y} = -6.216$$

$$u := \sqrt{U_{1x}^2 + U_{1y}^2} \qquad u = 6.284$$

$$\sigma := atan2(U_{1x}, U_{1y}) \qquad \sigma = -81.540 \, deg$$

6. Solve for links 3 and 1 using the vector definitions of **V** and **G**.

Link 3: $V_{1x} := z \cdot cos(\phi) - s \cdot cos(\psi)$ $V_{1x} = -3.660$

$V_{1y} := z \cdot sin(\phi) - s \cdot sin(\psi)$ $V_{1y} = 3.298$

$\theta_3 := atan2(V_{1x}, V_{1y})$ $\theta_3 = 137.980 \, deg$

$v := \sqrt{V_{1x}^2 + V_{1y}^2}$ $v = 4.927$

Link 1: $G_{1x} := w \cdot cos(\theta) + v \cdot cos(\theta_3) - u \cdot cos(\sigma)$ $G_{1x} = -4.133$

$G_{1y} := w \cdot sin(\theta) + v \cdot sin(\theta_3) - u \cdot sin(\sigma)$ $G_{1y} = 7.617$

$\theta_1 := atan2(G_{1x}, G_{1y})$ $\theta_1 = 118.485\,deg$

$g := \sqrt{G_{1x}^2 + G_{1y}^2}$ $g = 8.667$

7. Determine the initial and final values of the input crank with respect to the vector **G**.

$\theta_{2i} := \theta - \theta_1$ $\theta_{2i} = -195.092\,deg$

$\theta_{2f} := \theta_{2i} + \beta_2$ $\theta_{2f} = -165.092\,deg$

8. Define the coupler point with respect to point A and the vector **V**.

$r_p := z$ $\delta_p := \phi - \theta_3$

$r_p = 2.000$ $\delta_p = 12.020\,deg$

9. Locate the fixed pivots in the global frame using the vector definitions in Figure 5-2.

$\rho_1 := atan2(P_{1x}, P_{1y})$ $\rho_1 = 180.000\,deg$

$R_1 := \sqrt{P_{1x}^2 + P_{1y}^2}$ $R_1 = 0.000$

$O_{2x} := R_1 \cdot cos(\rho_1) - z \cdot cos(\phi) - w \cdot cos(\theta)$ $O_{2x} = 1.281$

$O_{2y} := R_1 \cdot sin(\rho_1) - z \cdot sin(\phi) - w \cdot sin(\theta)$ $O_{2y} = 0.896$

$O_{4x} := R_1 \cdot cos(\rho_1) - s \cdot cos(\psi) - u \cdot cos(\sigma)$ $O_{4x} = -2.853$

$O_{4y} := R_1 \cdot sin(\rho_1) - s \cdot sin(\psi) - u \cdot sin(\sigma)$ $O_{4y} = 8.514$

10. Determine the rotation angle of the fourbar frame with respect to the global frame (angle from the global X axis to the line O_2O_4.

$\theta_{rot} := atan2\left[(O_{4x} - O_{2x}), (O_{4y} - O_{2y})\right]$ $\theta_{rot} = 118.485\,deg$

11. Determine the Grashof condition.

$Condition(a, b, c, d) :=$
- $S \leftarrow min(a, b, c, d)$
- $L \leftarrow max(a, b, c, d)$
- $SL \leftarrow S + L$
- $PQ \leftarrow a + b + c + d - SL$
- *return* "Grashof" *if* $SL < PQ$
- *return* "Special Grashof" *if* $SL = PQ$
- *return* "non-Grashof" *otherwise*

$Condition(g, u, v, w) =$ "Grashof"

12. **DESIGN SUMMARY**

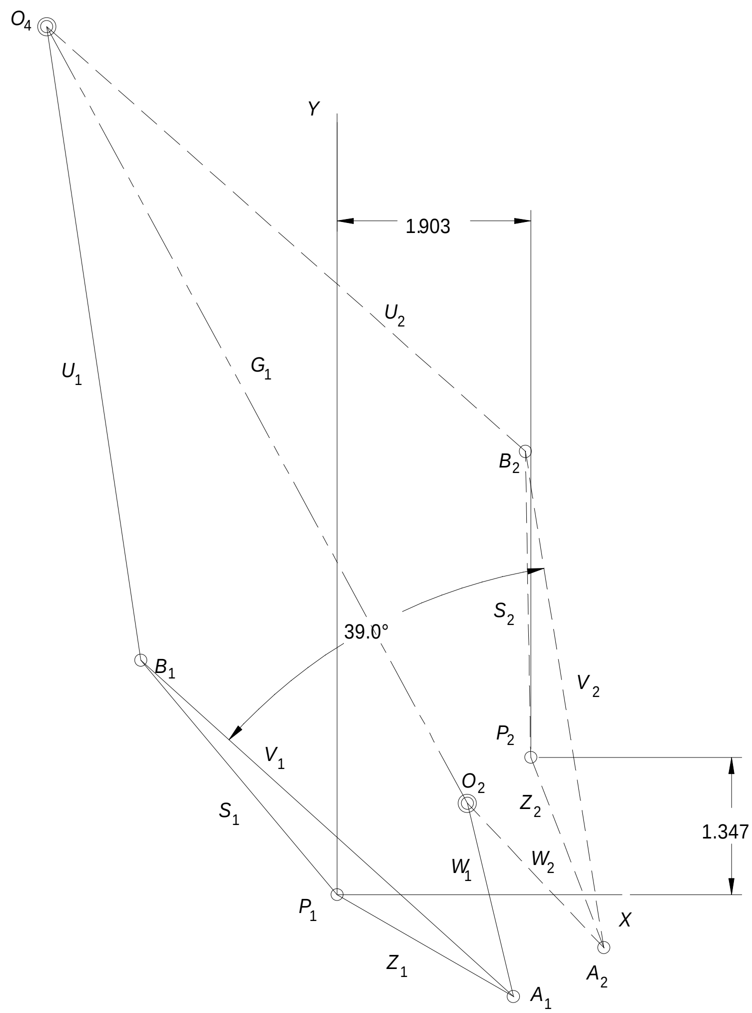
Link 2:	$w = 1.949$	$\theta = -76.607\,deg$
Link 3:	$v = 4.927$	$\theta_3 = 137.980\,deg$

Link 4:	$u = 6.284$	$\sigma = -81.540\ deg$
Link 1:	$g = 8.667$	$\theta_1 = 118.485\ deg$
Coupler:	$r_p = 2.000$	$\delta_p = 12.020\ deg$
Crank angles:		

$$\theta_{2i} = -195.092\ deg$$

$$\theta_{2f} = -165.092\ deg$$

13. Draw the linkage, using the link lengths, fixed pivot positions, and angles above, to verify the design.



PROBLEM 5-15

Statement: Design a linkage to carry the body in Figure P5-2 through the three positions P_1, P_2 and P_3 at the angles shown in the figure. Use analytical synthesis and design it for the fixed pivots shown.

Given:

$P_{21x} := 1.903$ $\quad P_{21y} := 1.347$ $\quad P_{31x} := 1.389$ $\quad P_{31y} := 1.830$

$O_{2x} := -0.884$ $\quad O_{2y} := -1.251$ $\quad O_{4x} := 3.062$ $\quad O_{4y} := -1.251$

Body angles: $\theta_{P1} := 101 \cdot deg$ $\quad \theta_{P2} := 62 \cdot deg$ $\quad \theta_{P3} := 39 \cdot deg$

Two argument inverse tangent

$$atan2(x,y) := \begin{cases} return\ 0.5\cdot\pi & if\ x = 0 \wedge y > 0 \\ return\ 1.5\cdot\pi & if\ x = 0 \wedge y < 0 \\ return\ atan\left(\left(\frac{y}{x}\right)\right) & if\ x > 0 \\ atan\left(\left(\frac{y}{x}\right)\right) + \pi & otherwise \end{cases}$$

Solution: See Figure P5-2 and Mathcad file P0515.

1. Determine the angle changes between precision points from the body angles given.

$\alpha_2 := \theta_{P2} - \theta_{P1}$ $\qquad \alpha_2 = -39.000\, deg$

$\alpha_3 := \theta_{P3} - \theta_{P1}$ $\qquad \alpha_3 = -62.000\, deg$

2. Using Figure 5-6, determine the magnitudes of $\mathbf{R}_1$, $\mathbf{R}_2$, and $\mathbf{R}_3$ and their x and y components.

$R_{1x} := -O_{2x}$ $\qquad R_{1x} = 0.884$ $\qquad R_{1y} := -O_{2y}$ $\qquad R_{1y} = 1.251$

$R_{2x} := R_{1x} + P_{21x}$ $\qquad R_{2x} = 2.787$

$R_{2y} := R_{1y} + P_{21y}$ $\qquad R_{2y} = 2.598$

$R_{3x} := R_{1x} + P_{31x}$ $\qquad R_{3x} = 2.273$

$R_{3y} := R_{1y} + P_{31y}$ $\qquad R_{3y} = 3.081$

$R_1 := \sqrt{R_{1x}^2 + R_{1y}^2}$ $\qquad R_1 = 1.532$

$R_2 := \sqrt{R_{2x}^2 + R_{2y}^2}$ $\qquad R_2 = 3.810$

$R_3 := \sqrt{R_{3x}^2 + R_{3y}^2}$ $\qquad R_3 = 3.829$

3. Using Figure 5-6, determine the angles that $\mathbf{R}_1$, $\mathbf{R}_2$, and $\mathbf{R}_3$ make with the x axis.

$\zeta_1 := atan2(R_{1x}, R_{1y})$ $\qquad \zeta_1 = 54.754\, deg$

$\zeta_2 := atan2(R_{2x}, R_{2y})$ $\qquad \zeta_2 = 42.990\, deg$

$\zeta_3 := atan2(R_{3x}, R_{3y})$ $\qquad \zeta_3 = 53.582\, deg$

4. Solve for β_2 and β_3 using equations 5.34

$C_1 := R_3 \cdot cos(\alpha_2 + \zeta_3) - R_2 \cdot cos(\alpha_3 + \zeta_2)$ $\qquad C_1 = 0.103$

$C_2 := R_3 \cdot sin(\alpha_2 + \zeta_3) - R_2 \cdot sin(\alpha_3 + \zeta_2)$ $C_2 = 2.205$

$C_3 := R_1 \cdot cos(\alpha_3 + \zeta_1) - R_3 \cdot cos(\zeta_3)$ $C_3 = -0.753$

$C_4 := -R_1 \cdot sin(\alpha_3 + \zeta_1) + R_3 \cdot sin(\zeta_3)$ $C_4 = 3.274$

$C_5 := R_1 \cdot cos(\alpha_2 + \zeta_1) - R_2 \cdot cos(\zeta_2)$ $C_5 = -1.313$

$C_6 := -R_1 \cdot sin(\alpha_2 + \zeta_1) + R_2 \cdot sin(\zeta_2)$ $C_6 = 2.182$

$A_1 := -C_3^{\,2} - C_4^{\,2}$ $A_1 = -11.288$

$A_2 := C_3 \cdot C_6 - C_4 \cdot C_5$ $A_2 = 2.654$

$A_3 := -C_4 \cdot C_6 - C_3 \cdot C_5$ $A_3 = -8.134$

$A_4 := C_2 \cdot C_3 + C_1 \cdot C_4$ $A_4 = -1.324$

$A_5 := C_4 \cdot C_5 - C_3 \cdot C_6$ $A_5 = -2.654$

$A_6 := C_1 \cdot C_3 - C_2 \cdot C_4$ $A_6 = -7.297$

$K_1 := A_2 \cdot A_4 + A_3 \cdot A_6$ $K_1 = 55.842$

$K_2 := A_3 \cdot A_4 + A_5 \cdot A_6$ $K_2 = 30.136$

$$K_3 := \frac{A_1^{\,2} - A_2^{\,2} - A_3^{\,2} - A_4^{\,2} - A_6^{\,2}}{2} \qquad K_3 = -0.392$$

$$\beta_{31} := 2 \cdot atan\left(\frac{K_2 + \sqrt{K_1^{\,2} + K_2^{\,2} - K_3^{\,2}}}{K_1 + K_3}\right) \qquad \beta_{31} = 118.708\,deg$$

$$\beta_{32} := 2 \cdot atan\left(\frac{K_2 - \sqrt{K_1^{\,2} + K_2^{\,2} - K_3^{\,2}}}{K_1 + K_3}\right) \qquad \beta_{32} = -62.000\,deg$$

The second value is the same as α_3, so use the first value $\beta_3 := \beta_{31}$

$$\beta_{21} := acos\left(\frac{A_5 \cdot sin(\beta_3) + A_3 \cdot cos(\beta_3) + A_6}{A_1}\right) \qquad \beta_{21} = 59.564\,deg$$

$$\beta_{22} := asin\left(\frac{A_3 \cdot sin(\beta_3) + A_2 \cdot cos(\beta_3) + A_4}{A_1}\right) \qquad \beta_{22} = 59.564\,deg$$

Since both values are the same, $\beta_2 := \beta_{21}$

5. Repeat steps 2, 3, and 4 for the right-hand dyad to find γ_1 and γ_2.

$R_{1x} := -O_{4x}$ $R_{1x} = -3.062$ $R_{1y} := -O_{4y}$ $R_{1y} = 1.251$

$R_{2x} := R_{1x} + P_{21x}$ $R_{2x} = -1.159$

$R_{2y} := R_{1y} + P_{21y}$ $R_{2y} = 2.598$

$R_{3x} := R_{1x} + P_{31x}$ $R_{3x} = -1.673$

$R_{3y} := R_{1y} + P_{31y}$ $R_{3y} = 3.081$

$R_1 := \sqrt{R_{1x}^2 + R_{1y}^2}$ $R_1 = 3.308$

$R_2 := \sqrt{R_{2x}^2 + R_{2y}^2}$ $R_2 = 2.845$

$R_3 := \sqrt{R_{3x}^2 + R_{3y}^2}$ $R_3 = 3.506$

6. Using Figure 5-6, determine the angles that $\mathbf{R}_1$, $\mathbf{R}_2$, and $\mathbf{R}_3$ make with the x axis.

$\zeta_1 := atan2(R_{1x}, R_{1y})$ $\zeta_1 = 157.777\ deg$

$\zeta_2 := atan2(R_{2x}, R_{2y})$ $\zeta_2 = 114.042\ deg$

$\zeta_3 := atan2(R_{3x}, R_{3y})$ $\zeta_3 = 118.502\ deg$

7. Solve for γ_2 and γ_3 using equations 5.34

$C_1 := R_3 \cdot cos(\alpha_2 + \zeta_3) - R_2 \cdot cos(\alpha_3 + \zeta_2)$ $C_1 = -1.111$

$C_2 := R_3 \cdot sin(\alpha_2 + \zeta_3) - R_2 \cdot sin(\alpha_3 + \zeta_2)$ $C_2 = 1.204$

$C_3 := R_1 \cdot cos(\alpha_3 + \zeta_1) - R_3 \cdot cos(\zeta_3)$ $C_3 = 1.340$

$C_4 := -R_1 \cdot sin(\alpha_3 + \zeta_1) + R_3 \cdot sin(\zeta_3)$ $C_4 = -0.210$

$C_5 := R_1 \cdot cos(\alpha_2 + \zeta_1) - R_2 \cdot cos(\zeta_2)$ $C_5 = -0.433$

$C_6 := -R_1 \cdot sin(\alpha_2 + \zeta_1) + R_2 \cdot sin(\zeta_2)$ $C_6 = -0.301$

$A_1 := -C_3^2 - C_4^2$ $A_1 = -1.840$

$A_2 := C_3 \cdot C_6 - C_4 \cdot C_5$ $A_2 = -0.495$

$A_3 := -C_4 \cdot C_6 - C_3 \cdot C_5$ $A_3 = 0.517$

$A_4 := C_2 \cdot C_3 + C_1 \cdot C_4$ $A_4 = 1.847$

$A_5 := C_4 \cdot C_5 - C_3 \cdot C_6$ $A_5 = 0.495$

$A_6 := C_1 \cdot C_3 - C_2 \cdot C_4$ $A_6 = -1.236$

$K_1 := A_2 \cdot A_4 + A_3 \cdot A_6$ $K_1 = -1.553$

$K_2 := A_3 \cdot A_4 + A_5 \cdot A_6$ $K_2 = 0.344$

$K_3 := \dfrac{A_1^2 - A_2^2 - A_3^2 - A_4^2 - A_6^2}{2}$ $K_3 = -1.033$

$\gamma_{31} := 2 \cdot atan\left(\dfrac{K_2 + \sqrt{K_1^2 + K_2^2 - K_3^2}}{K_1 + K_3}\right)$ $\gamma_{31} = -62.000\ deg$

$$\gamma_{32} := 2\cdot atan\left(\frac{K_2 - \sqrt{K_1^2 + K_2^2 - K_3^2}}{K_1 + K_3}\right) \qquad \gamma_{32} = 36.991\,deg$$

The first value is the same as α_3, so use the second value $\gamma_3 := \gamma_{32}$

$$\gamma_{21} := acos\left(\frac{A_5\cdot sin(\gamma_3) + A_3\cdot cos(\gamma_3) + A_6}{A_1}\right) \qquad \gamma_{21} = 73.415\,deg$$

$$\gamma_{22} := asin\left(\frac{A_3\cdot sin(\gamma_3) + A_2\cdot cos(\gamma_3) + A_4}{A_1}\right) \qquad \gamma_{22} = -73.415\,deg$$

Since γ_2 is not in the first quadrant , $\gamma_2 := \gamma_{22}$

8. Use the method of Section 5.7 to synthesize the linkage. Start by determining the magnitudes of the vectors $\mathbf{P}_{21}$ and $\mathbf{P}_{31}$ and their angles with respect to the X axis.

$$p_{21} := \sqrt{P_{21x}^2 + P_{21y}^2} \qquad p_{21} = 2.331$$

$$\delta_2 := atan2(P_{21x}, P_{21y}) \qquad \delta_2 = 35.292\,deg$$

$$p_{31} := \sqrt{P_{31x}^2 + P_{31y}^2} \qquad p_{31} = 2.297$$

$$\delta_3 := atan2(P_{31x}, P_{31y}) \qquad \delta_3 = 52.801\,deg$$

9. Evaluate terms in the **WZ** coefficient matrix and constant vector from equations (5.25) and form the matrix and vector:

$$A := cos(\beta_2) - 1 \qquad B := sin(\beta_2) \qquad C := cos(\alpha_2) - 1$$

$$D := sin(\alpha_2) \qquad E := p_{21}\cdot cos(\delta_2) \qquad F := cos(\beta_3) - 1$$

$$G := sin(\beta_3) \qquad H := cos(\alpha_3) - 1 \qquad K := sin(\alpha_3)$$

$$L := p_{31}\cdot cos(\delta_3) \qquad M := p_{21}\cdot sin(\delta_2) \qquad N := p_{31}\cdot sin(\delta_3)$$

$$AA := \begin{pmatrix} A & -B & C & -D \\ F & -G & H & -K \\ B & A & D & C \\ G & F & K & H \end{pmatrix} \qquad CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix} \qquad \begin{pmatrix} W1x \\ W1y \\ Z1x \\ Z1y \end{pmatrix} := AA^{-1}\cdot CC$$

10. The components of the **W** and **Z** vectors are:

$$W1x = 1.262 \qquad W1y = -1.109 \qquad Z1x = -0.378 \qquad Z1y = 2.360$$

11. The length of link 2 is: $w := \sqrt{W1x^2 + W1y^2}$ $\qquad w = 1.680$

12. Evaluate terms in the **US** coefficient matrix and constant vector from equations (5.25) and form the matrix and vector:

$A' := cos(\gamma_2) - 1$ $\quad B' := sin(\gamma_2)$ $\quad C := cos(\alpha_2) - 1$

$D := sin(\alpha_2)$ $\quad E := p_{21} \cdot cos(\delta_2)$ $\quad F' := cos(\gamma_3) - 1$

$G' := sin(\gamma_3)$ $\quad H := cos(\alpha_3) - 1$ $\quad K := sin(\alpha_3)$

$L := p_{31} \cdot cos(\delta_3)$ $\quad M := p_{21} \cdot sin(\delta_2)$ $\quad N := p_{31} \cdot sin(\delta_3)$

$$AA := \begin{pmatrix} A' & -B' & C & -D \\ F' & -G' & H & -K \\ B' & A' & D & C \\ G' & F' & K & H \end{pmatrix} \qquad CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix} \qquad \begin{pmatrix} U1x \\ U1y \\ S1x \\ S1y \end{pmatrix} := AA^{-1} \cdot CC$$

13. The components of the **W** and **Z** vectors are:

$U1x = -0.326$ $\quad U1y = 0.830$ $\quad S1x = -2.736$ $\quad S1y = 0.421$

14. The length of link 4 is: $u := \sqrt{U1x^2 + U1y^2}$ $\quad u = 0.892$

15. Solving for links 3 and 1 from equations 5.2a and 5.2b.

$V1x := Z1x - S1x$ $\quad V1x = 2.359$

$V1y := Z1y - S1y$ $\quad V1y = 1.939$

The length of link 3 is: $v := \sqrt{V1x^2 + V1y^2}$ $\quad v = 3.054$

$G1x := W1x + V1x - U1x$ $\quad G1x = 3.946$

$G1y := W1y + V1y - U1y$ $\quad G1y = -1.110 \times 10^{-15}$

The length of link 1 is: $g := \sqrt{G1x^2 + G1y^2}$ $\quad g = 3.946$

16. Check the location of the fixed pivots with respect to the global frame using the calculated vectors $\mathbf{W}_1$, $\mathbf{Z}_1$, $\mathbf{U}_1$, and $\mathbf{S}_1$.

$O2x := -Z1x - W1x$ $\quad O2x = -0.884$

$O2y := -Z1y - W1y$ $\quad O2y = -1.251$

$O4x := -S1x - U1x$ $\quad O4x = 3.062$

$O4y := -S1y - U1y$ $\quad O4y = -1.251$

These check with Figure P5-2.

17. Determine the location of the coupler point with respect to point *A* and line *AB*.

Distance from *A* to *P* $\quad z := \sqrt{Z1x^2 + Z1y^2}$ $\quad z = 2.390$ $\quad r_P := z$

Angle *BAP* (δ_p) $\quad s := \sqrt{S1x^2 + S1y^2}$ $\quad s = 2.769$

$\psi := atan2(S1x, S1y)$ $\quad \psi = 171.262\ deg$

$\phi := atan2(Z1x, Z1y)$ $\phi = 99.095\ deg$

$\theta_3 := atan2(z \cdot cos(\phi) - s \cdot cos(\psi), z \cdot sin(\phi) - s \cdot sin(\psi))$

$\theta_3 = 39.430\ deg$

$\delta_p := \phi - \theta_3$ $\delta_p = 59.666\ deg$

18. **DESIGN SUMMARY**

Link 1:	$g = 3.946$
Link 2:	$w = 1.680$
Link 3:	$v = 3.054$
Link 4:	$u = 0.892$
Coupler point:	$r_P = 2.390$ $\delta_p = 59.666\ deg$

19. **VERIFICATION**: The calculated values of g (length of the ground link) and of the coordinates of O_2 and O_4 give the same values as those on the problem statement, verifying that the calculated values for the other links and the coupler point are correct.